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THE LINDSTROM-MADDEN METHOD FOR SERIES SYSTEMS WITH REPEATED COMPONENTS

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ABSTRACT

The Lindstrom-Madden method of computing lower confidence limits for series systems with unlike components is extended to series systems with repeated components utilizing the results of Harris and Soms (1983). An exact solution is given for no failures and key test results, together with an approximation for the general case. Numerical examples are also provided.

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SIGNIFICANCE AND EXPLANATION

Series systems with repeated components arise often in engineering and physics. It is therefore important to utilize data obtained on individual components in an efficient manner when assessing the reliability of the combined system. This paper gives one method for doing so.

Accession For

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THE LINDSTROM-MADDEN METHOD FOR SERIES SYSTEMS WITH REPEATED COMPONENTS

Andrew P. Soms*

1. INTRODUCTION AND SUMMARY

A problem of substantial importance to practitioners in reliability is the statistical estimation of the reliability of a series system of stochastically independent components when some components are repeated, using experimental data collected on the individual components. In the situations discussed in this paper, the component data consist of a sequence of Bernoulli trials. Thus, for component i, $i = 1,2,\ldots,k$, the data is the pair (n_i,Y_i), where n_i is the number of trials and Y_i is the number of observations for which the component functions. $Y_1,Y_2,\ldots,Y_k$ are assumed to be mutually independent random variables. We assume that there are γ_i components of type i, $1 \leq i \leq k$. Then the parameter of interest is $h(p_1,p_2,\ldots,p_k) = h(\tilde{p})$, the reliability of the system, where

$$h(\tilde{p}) = \prod_{i=1}^{k} p_i^{\gamma_i} .$$

More specifically, it is desired to obtain a Buehler (1957) optimal lower $1 - \alpha$ confidence limit on $h(\tilde{p})$.

The case of $\gamma_1 = \gamma_2 = \cdots = \gamma_k = 1$ has been treated in Sudakov (1974), Winterbottom (1974), and Harris and Soms (1983).

In Section 2 we summarize the general theory of Harris and Soms (1983) applicable here. In Section 3 the exact solutions to no failures and key test results are given. Lindstrom-Madden type approximations are given in Section 4. Section 5 contains numerical examples.

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2. BUEHLER'S METHOD FOR OPTIMAL CONFIDENCE LIMITS

We now specialize the general results of Harris and Soms (1983) on optimal confidence limits for system reliability to a series system with independent and repeated components. As in Section 1, let

$$h(\tilde{p}) = \prod_{i=1}^{k} p_i^{Y_i} ,$$

$0 < p_i < 1$, $X_i = n_i - Y_i$, $x_i = n_i - y_i$, $1 \leq i \leq k$, $S = \{\tilde{x} | x_i = 0,1,\ldots,n_i,\ 1 \leq i \leq k\}$ and let $g(\tilde{x}) = (x_1,x_2,\ldots,x_k)$ be an ordering function, i.e., for real x_i, $0 \leq x_i \leq n_i$, $g(\tilde{x})$ is non-decreasing in each component. It is often convenient to normalize $g(\tilde{x})$ by letting $g(\tilde{0}) = 1$ and $g(\tilde{n}) = 0$. With such a normalization, $g(\tilde{x})$ is often selected to be a point estimator of $h(\tilde{p})$. Also let $R = \{r_1,r_2,\ldots,r_s,\ s \geq 2\}$ be the range set of $g(\tilde{x})$. With no loss of generality we order R so that $r_1 > r_2 > \ldots > r_s$ and let $A_i = \{\tilde{x} | g(\tilde{x}) = r_i,\ \tilde{x} \in S,\ i = 1,2,\ldots,s\}$. The sets A_i constitute a partition of S induced by $g(\tilde{x})$. We assume throughout that the data is distributed by

$$f(\tilde{x};\tilde{p}) = P_{\tilde{p}}(\tilde{X} = \tilde{x}) = \prod_{i=1}^{k} \binom{n_i}{x_i} p_i^{n_i - x_i} q_i^{x_i}$$

$$= \prod_{i=1}^{k} \binom{n_i}{y_i} p_i^{y_i} q_i^{n_i - y_i} , \tag{2.1}$$

where $q_i = 1 - p_i$, $i = 1,2,\ldots,k$. With no loss of generality, we assume $n_1 \leq n_2 \leq \ldots \leq n_k$.

From these definitions, it follows that

$$P_{\tilde{p}}\{X \in \bigcup_{i=1}^{j} A_i\} = P_{\tilde{p}}\{g(\tilde{X}) \geq r_j\} . \tag{2.2}$$

From (2.1) and (2.2), we have

$$P_{\tilde{p}}\{g(\tilde{X}) \geq r_j\} = \sum_{i_1=0}^{u_1} \sum_{i_2=0}^{u_2} \cdots \sum_{i_k=0}^{u_k} f(\tilde{i};\tilde{p}) , \tag{2.3}$$

where $\tilde{i} = (i_1, i_2, \ldots, i_k)$ and $u_2 = u_2(i_1), \ldots, u_k = u_k(i_1, i_2, \ldots, i_{k-1})$ are integers determined by r_j. Equivalently,

$$P_{\tilde{p}}\{g(\tilde{X}) \geq r_j\} = \sum_{i_1=0}^{[t_1]} \sum_{i_2=0}^{[t_2]} \cdots \sum_{i_k=0}^{[t_k]} f(\tilde{i};\tilde{p}) , \tag{2.4}$$

where $t_2 = t_2(i_1), \ldots, t_k = t_k(i_1, i_2, \ldots, i_{k-1})$, with $t_1 = \sup\{t | 0 \leq t \leq n_1$ and $g(t,0,0,\ldots,0) \geq r_j\}$ and $t_\ell(i_1, i_2, \ldots, i_{\ell-1}) = \sup\{t | 0 \leq t \leq n_\ell$ and $g(i_1, i_2, \ldots, i_{\ell-1}, t, 0, \ldots, 0) \geq r_j\}$, $\ell = 2,3,\ldots,k$.

We now introduce the notion of Buehler optimal confidence limits. Let $g(x) = r_j$. Then define

$$a_{g(\tilde{x})} = \inf\{h(\tilde{p}) | P_{\tilde{p}}\{\tilde{i} | g(\tilde{i}) \geq g(\tilde{x})\} \geq \alpha\} . \tag{2.5}$$

Equivalently, by (2.2), we can also write

$$a_{g(\tilde{x})} = \inf\left\{h(\tilde{p}) | P_{\tilde{p}}\left\{X \in \bigcup_{i=1}^{j} A_i\right\} \geq \alpha\right\} . \tag{2.6}$$

Then we have, from Harris and Soms (1983),

<u>Theorem 2.1</u>. $a_{g(\tilde{x})}$ is a $1 - \alpha$ lower confidence limit for $h(\tilde{p})$. If $b_{g(\tilde{X})}$ is any other $1 - \alpha$ lower confidence limit for $h(\tilde{p})$ with $b_{r_1} \geq b_{r_2} \geq \cdots \geq b_{r_j}$, then $b_{g(\tilde{x})} \leq a_{g(\tilde{x})}$ for all $\tilde{x} \in S$.

Two possible choices of $g(\tilde{x})$ are

$$g(\tilde{x}) = \prod_{i=1}^{k} ((n_i - x_i)/n_i)^{\gamma_i} , \tag{2.7}$$

or

$$g(\tilde{x}) = \prod_{i=1}^{k} \prod_{j=0}^{\gamma_i - 1} \left(\frac{n_i - x_i - j}{n_i - j}\right) . \tag{2.8}$$

Both reduce to the generally used $g(\tilde{x})$ for series systems with independent components when $\gamma_1 = \gamma_2 = \ldots = \gamma_k = 1$, i.e.,

$$g(\tilde{x}) = \prod_{i=1}^{k} (n_i - x_i)/n_i .$$

Since (2.7) is the maximum likelihood estimator of $h(\tilde{p})$ we will use it here and from now on it will be understood that $g(\tilde{x})$ is given by (2.7). With this choice of $g(\tilde{x})$, we assume from now on that $0 \leq x_i < n_i$, $i = 1,2,\ldots,k$, since $a_{g(\tilde{x})} = 0$ if some $x_i = n_i$. With this assumption, the t_i in (2.4) are given by

$$t_1 = n_1 - \left(\prod_{i=1}^{k} (n_i - x_i)^{\gamma_i} \Big/ \prod_{i=2}^{k} n_i^{\gamma_i}\right)^{1/\gamma_1} \tag{2.9}$$

and

$$t_\ell = n_1 - \left(\prod_{i=1}^{k} (n_i - x_i)^{\gamma_i} \Big/ \prod_{s=1}^{\ell-1} (n_s - i_s)^{\gamma_s} \prod_{i=\ell+1}^{k} n_i^{\gamma_i}\right)^{1/\gamma_\ell} , \tag{2.10}$$

$\ell = 2,\ldots,k$, with $\prod_{i=k+1}^{k} n_i^{\gamma_i} = 1$.

For the purpose of simplifying the calculation of $a_{g(\tilde{x})}$ in special cases it is necessary to state additional results from Harris and Soms (1983).

<u>Theorem 2.2</u>. Let $g(\tilde{x}) = r_j$ and let

$$f^*(x;a) = \sup_{h(\tilde{p})=a} P_{\tilde{p}}\{g(\tilde{X}) \geq r_j\}, \quad 0 < a < 1 . \tag{2.11}$$

Then

$$\inf_{0<a<1} f^*(\tilde{x};a) = 0, \qquad \sup_{0<a<1} f^*(\tilde{x};a) = 1$$

and $f^*(\tilde{x};a)$ is strictly increasing in a.

Theorem 2.3. $f^*(\tilde{x};a) = \alpha$ has exactly one solution a_α in a and $a_\alpha = a_{g(\tilde{x})}$.

3. EXACT SOLUTIONS FOR ZERO FAILURES AND KEY TEST RESULTS

We first assume that $\tilde{x} = (0,0,\ldots,0) = \tilde{0}$ and use Theorem 2.3 to obtain $a_{g(\tilde{0})}$.

Theorem 3.1. If $\tilde{x} = \tilde{0}$, then

$$f^*(\tilde{0};a) = \sup_{\prod_{i=1}^k p_i^{\gamma_i}=a} \prod_{i=1}^k p_i^{n_i} = a^{n_j/\gamma_j}, \tag{3.1}$$

where $n_j/\gamma_j = \min_{1\leq i\leq k} n_i/\gamma_i$ and

$$a_{g(\tilde{0})} = \alpha^{\gamma_j/n_j}. \tag{3.2}$$

Proof.

$$\prod_{i=1}^k p_i^{n_i} = \left(\prod_{i=1}^k p_i^{\gamma_i}\right)^{n_j/\gamma_j} \prod_{\substack{i=1\\ i\neq j}}^k p_i^{(n_i\gamma_j - n_j\gamma_i)/\gamma_j}$$
$$\leq a^{n_j/\gamma_j},$$

since $n_i\gamma_j - n_j\gamma_i \geq 0$ is equivalent to $n_i/\gamma_i \geq n_j/\gamma_j$, which is true, and therefore $\prod_{\substack{i=1\\ i\neq j}}^k p_i^{(n_i\gamma_j - n_j\gamma_i)/\gamma_j} \leq 1$. (3.1) follows by noting that the choice $p_j = a^{1/\gamma_j}$, $p_i = 1$, $i \neq j$, gives $\prod_{i=1}^k p_i^{n_i} = a^{n_j/\gamma_j}$. Then, using Theorem 2.3, we obtain (3.2), which reduces to the known series result if $\gamma_1 = \gamma_2 = \ldots = \gamma_k = 1$.

We now turn to analogues of key test results (see, e.g., Winterbottom (1974) and Harris and Soms (1983)). We define a key test result if $\gamma_1 = \max_{1 \le i \le k} \gamma_i$ (recall that $n_1 = \min_{1 \le i \le k} n_i$) and $\tilde{x} = (x_1, 0, \ldots, 0)$.

<u>Theorem 3.2</u>. If $\tilde{x}$ is a key test result and

$$\{\tilde{z} \mid \prod_{i=1}^{k} (n_i - z_i)^{\gamma_i} \ge \prod_{i=1}^{k} (n_i - x_i)^{\gamma_i}\} = \{\tilde{z} \mid \sum_{i=1}^{k} (n_i - z_i) \ge \sum_{i=1}^{k} (n_i - x_i)\} , \tag{3.3}$$

then

$$f^*(\tilde{x}; a) = I_{a^{1/\gamma_1}}(n - x_1, x_1 + 1) , \tag{3.4}$$

where $I_x(a,b)$ is the incomplete beta *function. Let* b_α *denote the solution in* b *of*

$$\alpha = I_b(n_1 - x_1, x_1 + 1) .$$

Then $a_{g(\tilde{x})} = b_\alpha^{\gamma_1}$. Note that b_α is the usual $1 - \alpha$ lower confidence limit on p, *given* x_1 *failures in* n_1 *trials.*

<u>Proof</u>. Without loss of generality we can assume that $n_1 = n_2 = \ldots = n_k$, for otherwise we can write (2.4) as

$$P_{\tilde{p}}\{g(\tilde{X}) \ge r_j\} = \sum_{i_1=0}^{x_1} \binom{n_1}{i_1} p_1^{n_1-i_1} q_1^{i_1} \sum_{i_2=0}^{x_1-i_1} \binom{n_2}{i_2} p_2^{n_2-i_2} q_2^{i_2} \cdots$$

$$\sum_{i_{k-1}=0}^{x_1-i_1-i_2-\ldots-i_{k-2}} \binom{n_{k-1}}{i_{k-1}} p_{k-1}^{n_{k-1}-i_{k-1}} q_{k-1}^{i_{k-1}} I_{p_k}(n_k -$$

$$(x_1-i_1-i_2-\ldots-i_{k-1}), x_1-i_1-i_2-\ldots-i_{k-1}+1)$$

$$\le \sum_{i_1=0}^{x_1} \binom{n_1}{i_1} p_1^{n_1-i_1} q_1^{i_1} \cdots \sum_{i_{k-1}=0}^{x_1-i_1-i_2\ldots-i_{k-2}} \binom{n_{k-1}}{i_{k-1}} p_{k-1}^{n_{k-1}-i_{k-1}} q_{k-1}^{i_{k-1}} I_{p_k}(n_1 -$$

$$(x_1-i_1-i_2-\ldots-i_{k-1}), x_1-i_1-i_2\ldots-i_{k-1}+1) , \tag{3.5}$$

where $g(\tilde{x}) = r_j$, by the monotone likelihood ratio property of the beta distribution ($I_x(a,b)$ has a monotone likelihood ratio in $-a$ for fixed b, which implies that $I_x(a,b)$ is a decreasing function of a). A similar argument applies to the other indexes. Thus, if (3.4) is true for $n_1 = n_2 = \cdots = n_k$, by (3.5) it follows for $n_1 \leq n_2 \leq \cdots \leq n_k$.

So, assuming $\tilde{n} = (n_1, n_1, \ldots, n_1)$, we seek to maximize

$$P_{\tilde{p}}\left\{\sum_{i=1}^{k} \sum_{j=1}^{n_1} Y_{ij} \geq \sum_{i=1}^{k} (n_i - x_i) = \sum_{i=1}^{k} y_i\right\}, \tag{3.6}$$

where Y_{ij} are independent Bernoulli random variables with parameter p_i and $\prod_{i=1}^{k} p_i^{\gamma_i} = a$. If $\prod_{i=1}^{k} p_i^{\gamma_i} = a$, then $\prod_{i=1}^{k} p_i$ ranges from a^{1/γ_j} to a^{1/γ_1}, $\gamma_j = \min_{1 \leq i \leq k} \gamma_i$. This is seen as follows:

$$\prod_{i=1}^{k} p_i = \left(\prod_{i=1}^{k} p_i^{\gamma_i}\right)^{1/\gamma_1} \prod_{i=2}^{k} p_i^{1-\gamma_i/\gamma_1}$$

$$= a^{1/\gamma_1} \prod_{i=2}^{k} p_i^{(\gamma_1-\gamma_i)/\gamma_1} \leq a^{1/\gamma_1}$$

and

$$\prod_{i=1}^{k} p_i = \left(\prod_{i=1}^{k} p_i^{\gamma_i}\right)^{1/\gamma_j} \prod_{\substack{i=1 \\ i \neq j}}^{k} p_i^{1-\gamma_i/\gamma_j}$$

$$= a^{1/\gamma_j} \prod_{\substack{i=1 \\ i \neq j}}^{k} p_i^{(\gamma_j-\gamma_i)/\gamma_j} \geq a^{1/\gamma_j}$$

and the choices $p_1 = a^{1/\gamma_1}$, $p_2 = \cdots = p_k = 1$, and $p_j = a^{1/\gamma_j}$, $p_i = 1,\ i \neq j$, attain these values. From the results of Pledger and Proschan (1971), for each $b = \prod_{i=1}^{k} p_i$,

$a^{1/\gamma_j} \le b \le a^{1/\gamma_1}$, (3.6) is maximized by $p_1 = b$, $p_i = 1$, $2 \le i \le k$. Further, the maximum over b, $a^{1/\gamma_j} \le b \le a^{1/\gamma_1}$, of the maxima for each b is given by $p_1 = a^{1/\gamma_1}$, $p_i = 1$, $2 \le i \le k$, by the monotone likelihood ratio property of the binomial distribution, and $p_1 = a^{1/\gamma_1}$, $p_i = 1$, $2 \le i \le k$, satisfies $\prod_{i=1}^{k} p_i^{\gamma_i} = a$. This completes the proof.

If $\gamma_1 = \gamma_2 = \cdots = \gamma_k = 1$, some guidelines for the verification of (3.3) are given in Harris and Soms (1983). In the present case (3.3) must be verified by trial and error by showing that

$$\min_{\sum_{i=1}^{k} x_i = x_1} \prod_{i=1}^{k} (n_i - x_i)^{\gamma_i} = (n_1 - x_1)^{\gamma_1} \prod_{i=2}^{k} n_i^{\gamma_i} \quad \text{and that}$$

$$\max_{\sum_{i=1}^{k} x_i = x_1+1} \prod_{i=1}^{k} (n_i - x_i)^{\gamma_i} < (n_1 - x_1)^{\gamma_1} \prod_{i=2}^{k} n_i^{\gamma_i}.$$

Example 3.1. Let $k = 3$, $\tilde{n} = (5,5,5)$, $\tilde{\gamma} = (3,3,2)$, $\alpha = .10$ and $\tilde{x} = (1,0,0)$. Then

$$\min_{\sum_{i=1}^{3} x_i = 1} \prod_{i=1}^{3} (n_i - x_i)^{\gamma_i} = 200000 \quad \text{and} \quad \max_{\sum_{i=1}^{3} x_i = 2} \prod_{i=1}^{3} (n_i - x_i)^{\gamma_i} = 140625 \quad \text{and}$$

$\tilde{x}$ is a key test result and (3.3) is satisfied and hence

$$a_{g(\tilde{x})} = .4161^3 = .0720 ,$$

where $.10 = I_{.4161}(4,2)$. Further, it can also be verified that $\tilde{x} = (2,0,0)$ is a key test result for which (3.3) is satisfied, but that for $\tilde{x} = (3,0,0)$, (3.3) is violated.

Note that Theorem 3.2 asserts that $a_{g(\tilde{x})} = b_\alpha^{\gamma_1}$ for $0 < \alpha < 1$. It is thus possible that (3.3) is not true but the conclusion still holds for α of practical importance. This is taken up in Section 4.

4. THE LINDSTROM-MADDEN METHOD FOR SERIES SYSTEMS WITH REPEATED COMPONENTS

When $\gamma_1 = \gamma_2 = \cdots = \gamma_r = 1$, the Lindstrom-Madden method (henceforth abbreviated L-M) is an approximation $b_{g(\tilde{x})}$ to $a_{g(\tilde{x})}$ of the form

$$b_{g(\tilde{x})} = \min_{1 \le i \le k} b_\alpha(n_i) , \tag{4.1}$$

where

$$\alpha = I_{b_\alpha(n_i)}(n_i - t_{0i}, t_{0i} + 1) , \tag{4.2}$$

with

$$t_{0i} = n_i(1 - \prod_{i=1}^{k} (n_i - x_i)/n_i) , \tag{4.3}$$

i.e., t_{0i} is the maximum of the recursive indexes t_i defined by (2.4). For the usual levels of α, $b_{g(\tilde{x})} = b_\alpha(n_1)$. Further, numerical evidence indicates (Harris and Soms (1983)) that for α levels of practical significance

$$b_{g(\tilde{x})} \leq a_{g(\tilde{x})} . \tag{4.4}$$

(4.4) was incorrectly claimed to be true for $0 < \alpha < 1$ in Sudakov (1974) and this is discussed at length in Harris and Soms (1983). However, (4.4) is known to hold for special cases (Winterbottom (1974) and Harris and Soms (1983)).

Motivated by the above, we now give an L-M approximation $b_{g(\tilde{x})}$ to $a_{g(\tilde{x})}$ for arbitrary γ_i by

$$b_{g(\tilde{x})} = \min_{1 \leq i \leq k} b_\alpha(n_i)^{\gamma_i} , \tag{4.5}$$

where

$$\alpha = I_{b_\alpha(n_i)}(n_i - t_{0i}, t_{0i} + 1) , \tag{4.6}$$

with

$$t_{0i} = n_i - (\prod_{j=1}^{k} (n_j - x_j)^{\gamma_j} \Big/ \prod_{\substack{j=1 \\ j \neq i}}^{k} n_j^{\gamma_j})^{1/\gamma_i} , \tag{4.7}$$

i.e., t_{0i} is the maximum of the recursive indexes t_i defined by (2.4). However, in this case it is not clear which index i gives the minimum, except that the likely candidate is the one for which γ_j, $1 \le j \le k$, is a maximum. We might expect, by analogy, that for α levels of practical interest

$$b_{g(\tilde{x})} \le a_{g(\tilde{x})} . \tag{4.8}$$

5. NUMERICAL EXAMPLES

For $k = 2$ and selected $\tilde{n}$, $\tilde{\gamma}$, $\tilde{x}$, $\alpha = .05$ and $.10$, Table I gives $b_{g(\tilde{x})}$, $a_{g(\tilde{x})}$ and the best upper bound, $u_{g(\tilde{x})}$,

$$u_{g(\tilde{x})} = \min_{1 \le i \le k} u_\alpha(n_i)^{\gamma_i} , \tag{5.1}$$

where

$$\alpha = I_{u_\alpha(n_i)}(n_i - [t_{0i}], [t_{0i}] + 1) \tag{5.2}$$

and t_{0i} are defined as in (4.6).

TABLE I.

L-M Approximations and $a_{g(\tilde{x})}$

(n_1,n_2)	(γ_1,γ_2)	(x_1,x_2)	α	$b_{g(\tilde{x})}$	$a_{g(\tilde{x})}$	$u_{g(\tilde{x})}$
(10,10)	(1,2)	(0,1)	.05	.3670	.3670	.3670
(10,10)	(1,2)	(0,1)	.10	.4398	.4398	.4398
(10,10)	(1,2)	(1,1)	.05	.3045	.3514	.3670
(10,10)	(1,2)	(1,1)	.10	.3715	.4227	.4398
(10,10)	(1,2)	(2,1)	.05	.2484	.3151	.3670
(10,10)	(1,2)	(2,1)	.10	.3088	.3825	.4398
(10,15)	(2,3)	(0,1)	.05	.3695	.3719	.3742
(10,15)	(2,3)	(0,1)	.10	.4425	.4446	.4467
(10,15)	(2,3)	(1,1)	.05	.2554	.3042	.3670
(10,15)	(2,3)	(1,1)	.10	.3167	.3705	.4398
(10,15)	(2,3)	(2,1)	.05	.1712	.1981	.2431
(10,15)	(2,3)	(2,0)	.10	.2203	.2513	.3029

Note that for all the cases in Table I, $b_{g(\hat{x})}$ is a lower bound for $a_{g(\hat{x})}$. The computations were done by a short FORTRAN program, a listing of which can be obtained from the author.

6. CONCLUDING REMARKS

In this paper we have extended the L-M method to series systems with repeated components. More work is needed to ascertain the region of validity of (4.8).

BIBLIOGRAPHY

Buehler, R. J., (1957). Confidence Limits for the Product of Two Binomial Parameters. Journal of the American Statistical Association, 52, 482-93.

Harris, B. and Soms, A. P., (1983). The Theory of Optimal Confidence Limits for Systems Reliability with Counterexamples for Results on Optimal Confidence Limits for Series Systems. Technical Report #708, Department of Statistics, University of Wisconsin-Madison.

Pledger, G. and Proschan, F., (1971). Comparison of Order Statistics and of Spacings from Heterogeneous Distributions. Optimizing Methods in Statistics, New York: Academic Press, 89-113.

Sudakov, R. S., (1974). On the Question of Interval Estimation of the Index of Reliability of a Sequential System. Engineering Cybernetics, 12, 55-63.

Winterbottom, A., (1974). Lower Limits for Series System Reliability from Binomial Data. Journal of the American Statistical Association, 69, 782-8.

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The Lindstrom-Madden method of computing lower confidence limits for series systems with unlike components is extended to series systems with repeated components utilizing the results of Harris and Soms (1983). An exact solution is given for no failures and key test results, together with an approximation for the general case. Numerical examples are also provided.

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